# A Low Reynolds Number Discharge Coefficient Equation for Critical Flow Venturi Nozzles and the Effects of Inlet Radius

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# Outline of Presentation

- **Introduction**
- **Development of a Low Reynold Number Empirical Equation**
  - Reduction of calibration data
  - Data correction for throat diameter
  - Curve fit generation
  - Uncertainty calculation
- **Inlet Curvature Testing**
- **Conclusions**

# Introduction

- Critical Flow Venturi Nozzles (CFVNs) are used in a variety of gas flow measurement applications.
- The theoretical mass flow through a CFVN is calculated using 1-D isentropic theory which does not account for the subsonic boundary layer.
- The Discharge Coefficient (Cd) is primarily used to correct for this boundary layer.
- The effect on Cd due to the boundary layer is more significant for smaller throat CFVNs and therefore for low Reynold Number CFVN applications.

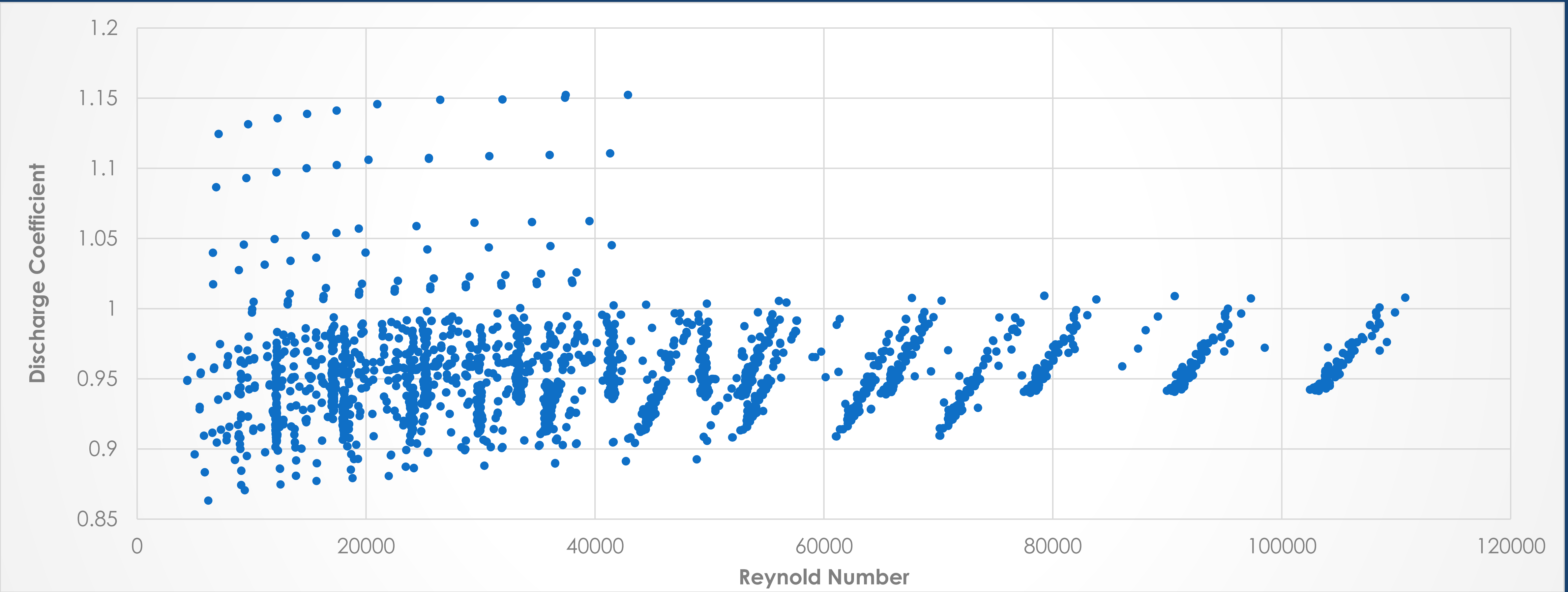
# Introduction

- The international standard governing CFVNs provides an Empirical Cd-Re equation that can be used down to a minimum Re of 21000.
- An Empirical equation for use below a Re of 21000 is needed for low Re mass flow or sizing calculation for CFVNs.
- Inlet curvature is difficult to control when manufacturing small CFVNs.
- The ISO inlet curvature requirement of 1.8-2.2 times the throat diameter needs to be evaluated for low Re applications.

# Collected Calibration Data

- Calibration Data take over 5 years for 184 CFVNs
- Total of 3613 data points
- Throat diameters varied from 0.28mm - 2.36mm
- Geometry was per ISO 9300
- Uncertainty in all Cd values was 0.2% or less (k=2)
- Only calibrations that included data both above and below a Reynold Number of 21000 were used

# Collected Calibration Data



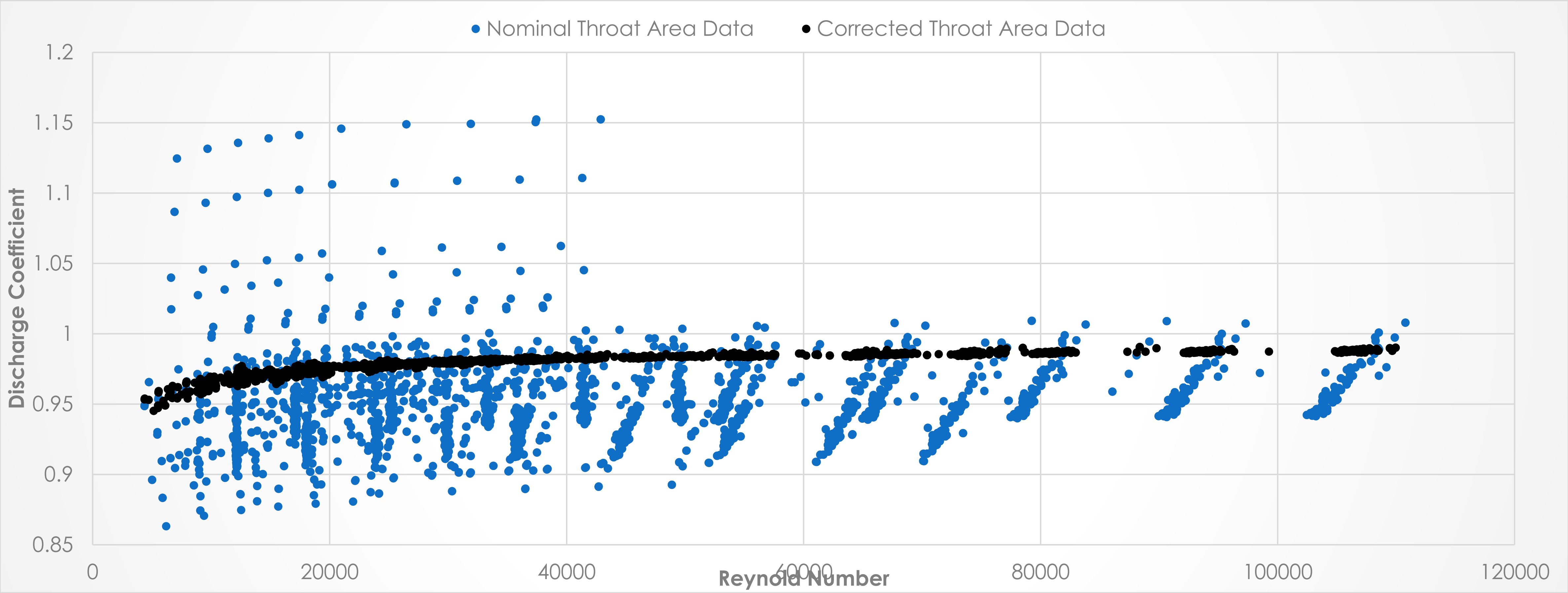
# Reduction of Calibration Data

- The large scatter in the data is due to the "nominal" throat diameter being used and not the actual throat diameter
- For typical CFVN operation this is acceptable as the "nominal" value is used in the calibration and operation and is therefore fully correlated
- In order to generate a low Re empirical equation the data needs to be corrected to use actual throat diameter

# Reduction of Calibration Data

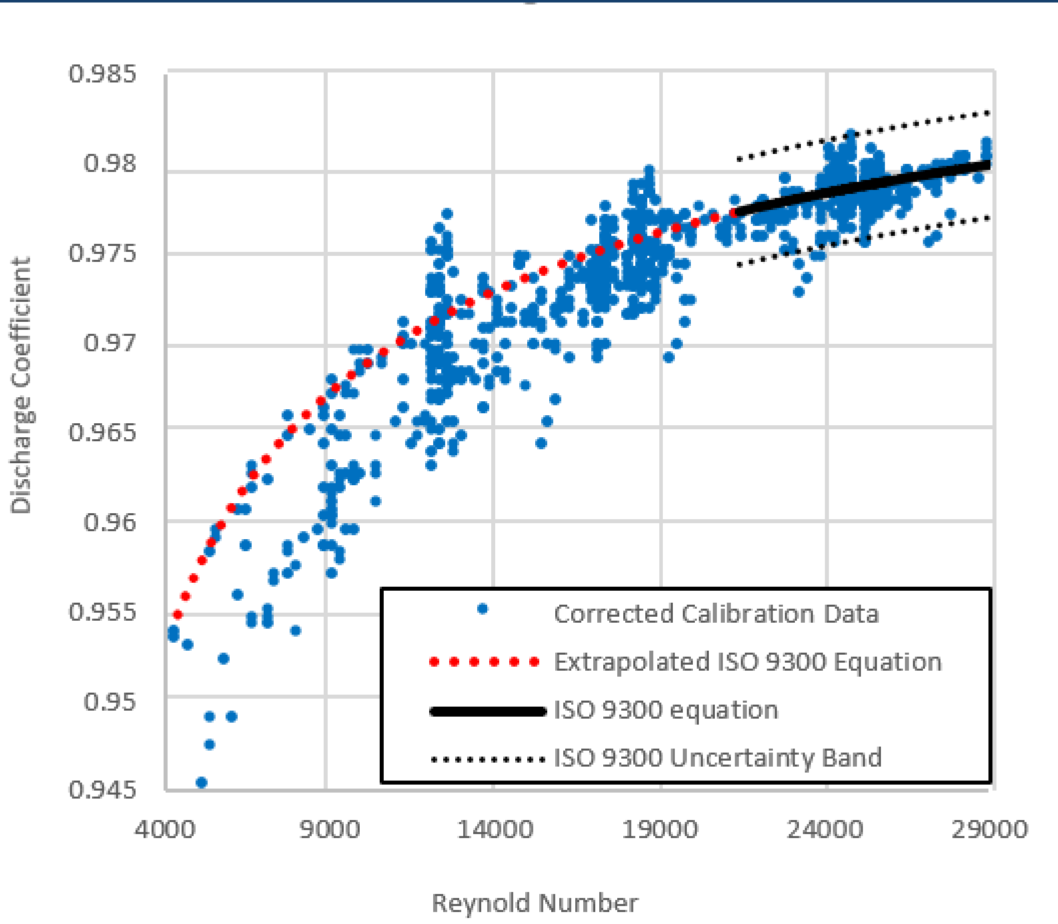
- For each calibration the data collected at a Re greater than 21000 was compared to the Cd value predicted by the ISO Empirical Equation
- The ratio of the calibration Cd to the ISO Empirical Equation Cd was then used as multiplier on the nominal throat area to calculate the new throat area and therefore the new throat diameter
- The calibration data was then re-processed with the new throat diameters and this process was iterated until the values converged to an actual throat diameter with Cd and Reynolds values based on that throat diameter

# Reduction of Calibration Data



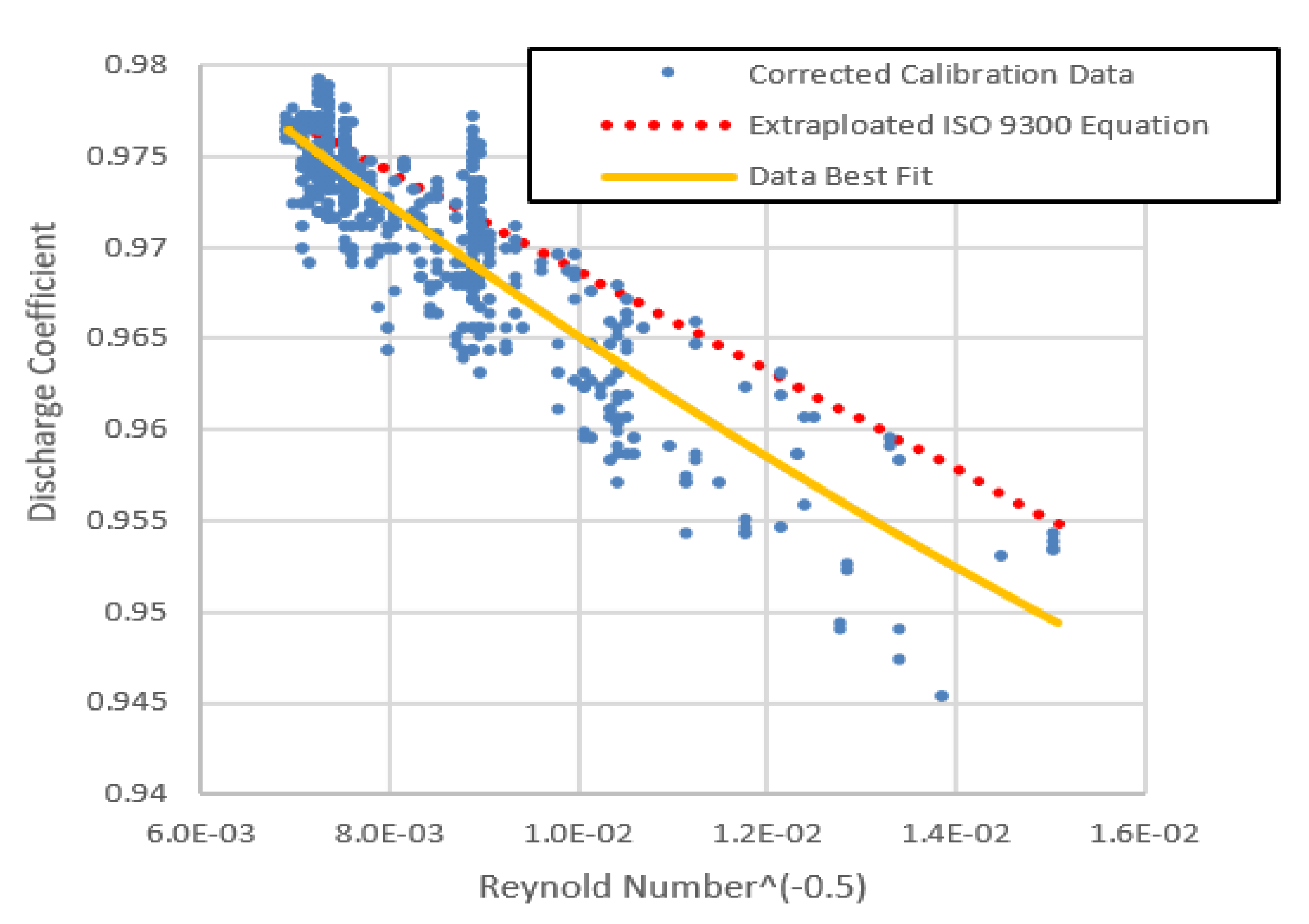
# Comparison to ISO 9300 Equation

- The re-processed data was then compared to the extrapolated ISO 9300
- The extrapolated ISO equation over predicts Cd by 0.5-1.0%
- A new low Re Empirical Equation is necessary



# Low Reynold Number Curve Fit

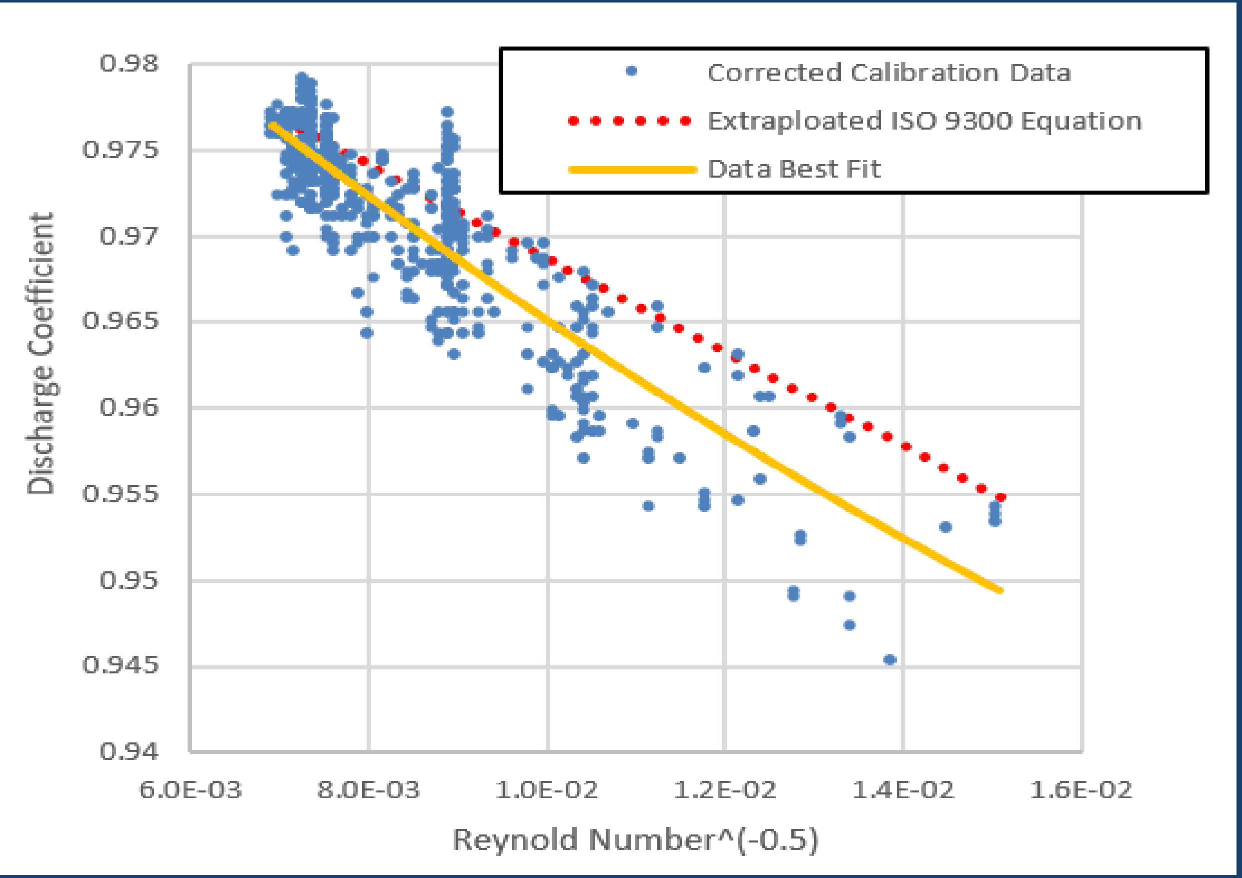
- To generate the best fit equation Cd was plotted against Re^(-0.5) as this partially linearizes the result



# Low Reynold Number Curve Fit

$$Cd = 1.0068 - 4.8720 \times Re^{-0.5} + 70.895 \times Re^{-1}$$

$$7000 \leq Re \geq 21000$$



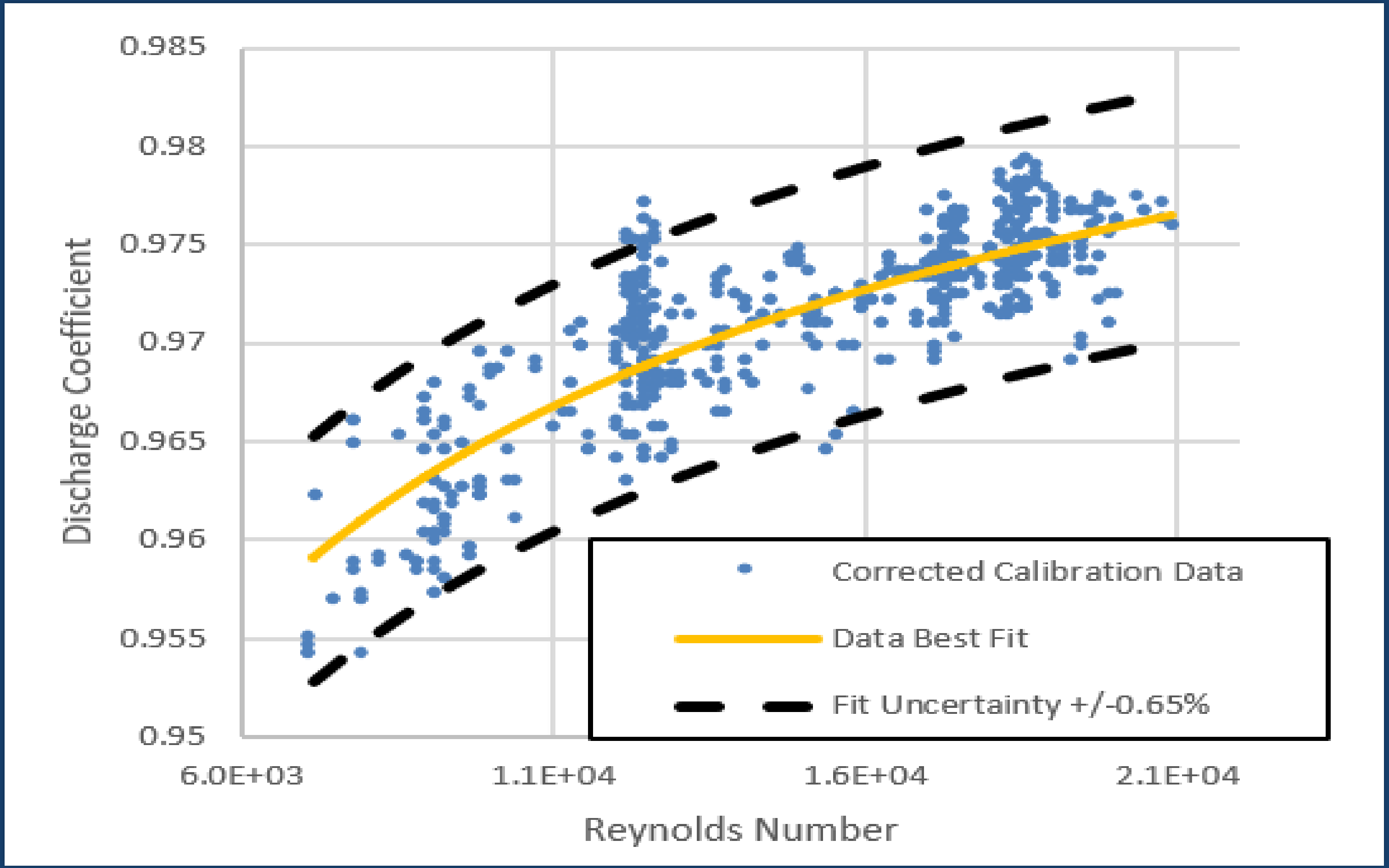
# Curve Fit Uncertainty

- Three components to calculate the uncertainty of the Empirical Equation
  - Cd uncertainty from each of the 184 CFVN Calibrations (0.2%)
  - Cd uncertainty in the ISO 9300 equation used to correct throat diameters (0.3%)
  - Residuals on the curve fit equation (+/- band on curve fit that contains 95% of the data)
- Three components were combined using the RSS method

$$U = \left[U_c{}^2 + U_d{}^2 + U_f{}^2\right]^{0.5}$$

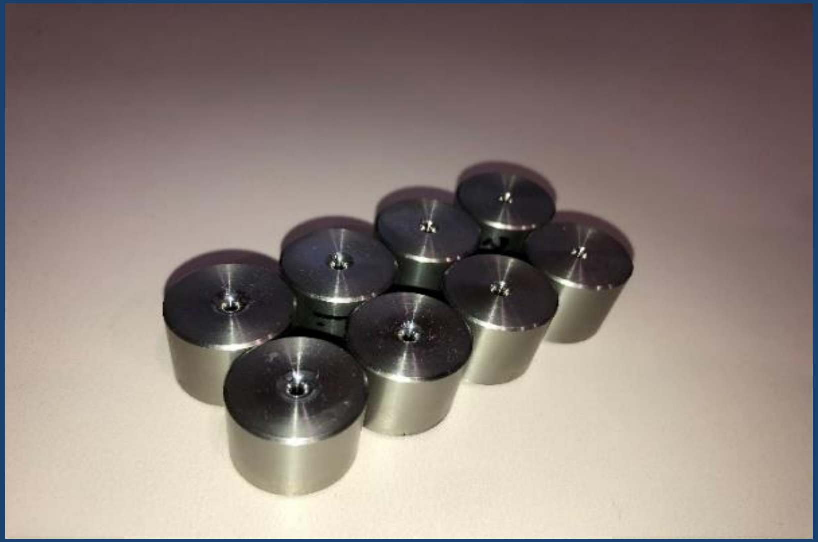
- Resulting combined uncertainty of 0.65% (k=2) on the Empirical Equation

# Empirical Equation

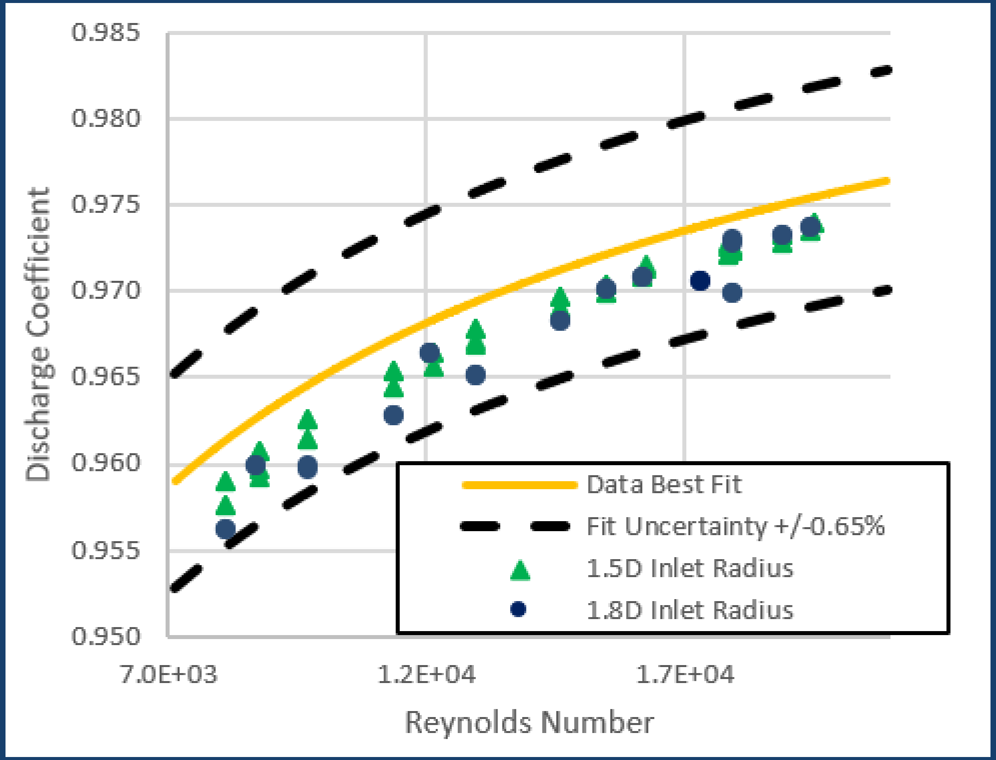


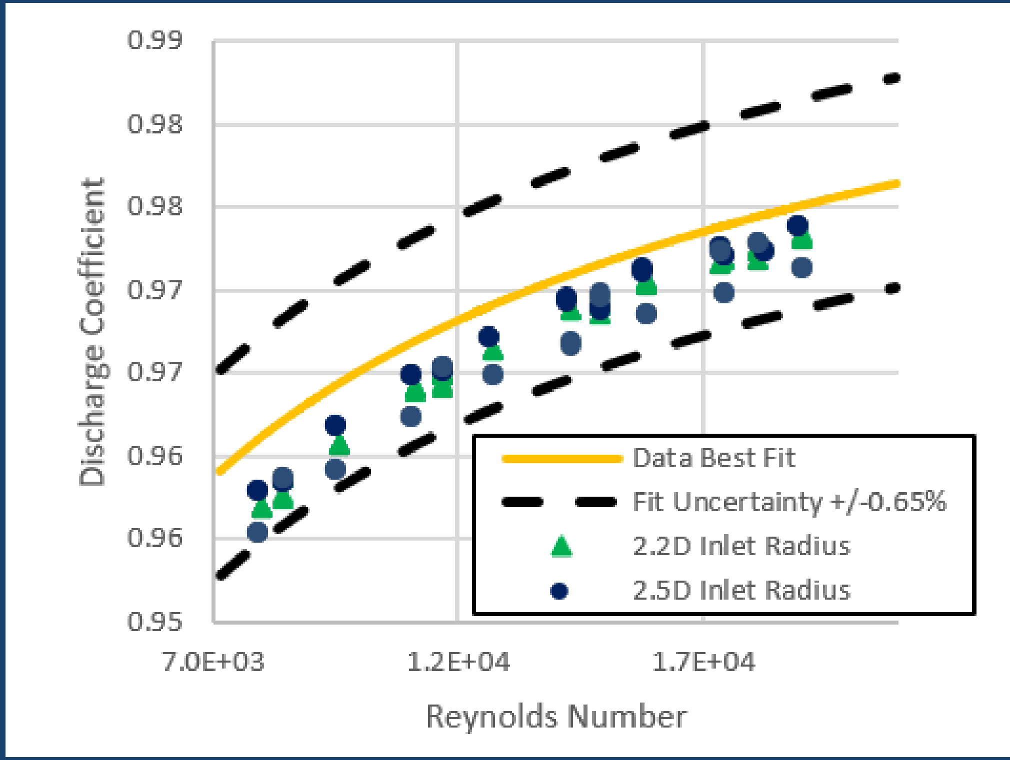
# Inlet Curvature Effects on Cd

- All the CFVNs used to establish the low Re Empirical Equation had nominally 2d inlet curvature but due to manufacturing variations it was hypothesized that the inlet curvature may have caused the scatter observed in the throat corrected data.
- To Test this, 8 new 1.47mm throat CFVNs were manufactured with inlet curvatures of 1.5d, 1.8d, 2.2d, and 2.5d to test the effect on Cd



# Inlet Curvature Effects on Cd



# Inlet Curvature Effects on Cd

- All the 1.47mm CFVN data falls with the uncertainty bands on the Empirical Equation
- Theory predicts that below a Re of 150000 decreased inlet curvature should result in increased Cd values
- The 1.5d and 1.8d CFVNs show a behavior that contradicts this theory
- This suggests that the data scatter seen in not due to inlet curvature but may be due to another factor such as surface finish or the transition from the inlet curvature to the diffuser cone

# Conclusions

- From the reduction of calibration data from 184 CFVNs a new Low Reynolds Number Empirical Equation has been developed:

$$Cd = 1.0068 - 4.8720 \times Re^{-0.5} + 70.895 \times Re^{-1}$$

$$7000 \leq Re \geq 21000$$

uncertainty of 0.65% $(k = 2)$

- The equation is valid for CFVNs built with the geometry outlined in ISO 9300 and seems to allow for inlet curvature from 1.5d to 2.5d

# The End

- Thank you for your attention.

- Questions?